On the Use of Manufactured Solutions for Code Verification of RANS Solvers Based on Eddy-viscosity Models

L. Eça (TU-Lisbon, IST)
M. Hoekstra (MARIN)
G. Vaz (MARIN)

Contents

1. Introduction

- Assess the (discretization) convergence properties of RANS solvers based on eddy-viscosity models
- Determination of discretization error requires analytical (exact) solutions
- Manufactured Solutions that resemble real turbulent flows
- Present examples are for the Spalart & Allmaras one-equation model and for the TNT version of the k-ω two-equation model

2. Manufactured Solutions

- The flow field is defined as a function of the Reynolds number, allowing the choice of values in the range of 10^6 to 10^9, $Re = \frac{U_1 L}{\nu}$

- Bottom boundary of the domain is a “wall”

- Velocity field is divergence free

- Mean velocity profiles include a “viscous sub-layer” in the near wall region

2. Manufactured Solutions

- Skin-friction coefficient matches an empirical correlation for a flat plate boundary-layer

$$C_f = \frac{\tau_w}{1/2\rho U_1^2} = 0.058\left(Re_x\right)^{-0.2}, \quad Re_x = Re\frac{x}{L}$$

- Flow field tends to a uniform flow with the increase of the “distance to the wall”
- Alternative MS’s obtained from superposition with a perturbation flow that does not change the near wall region
- Pressure field matches typical boundary conditions of practical applications

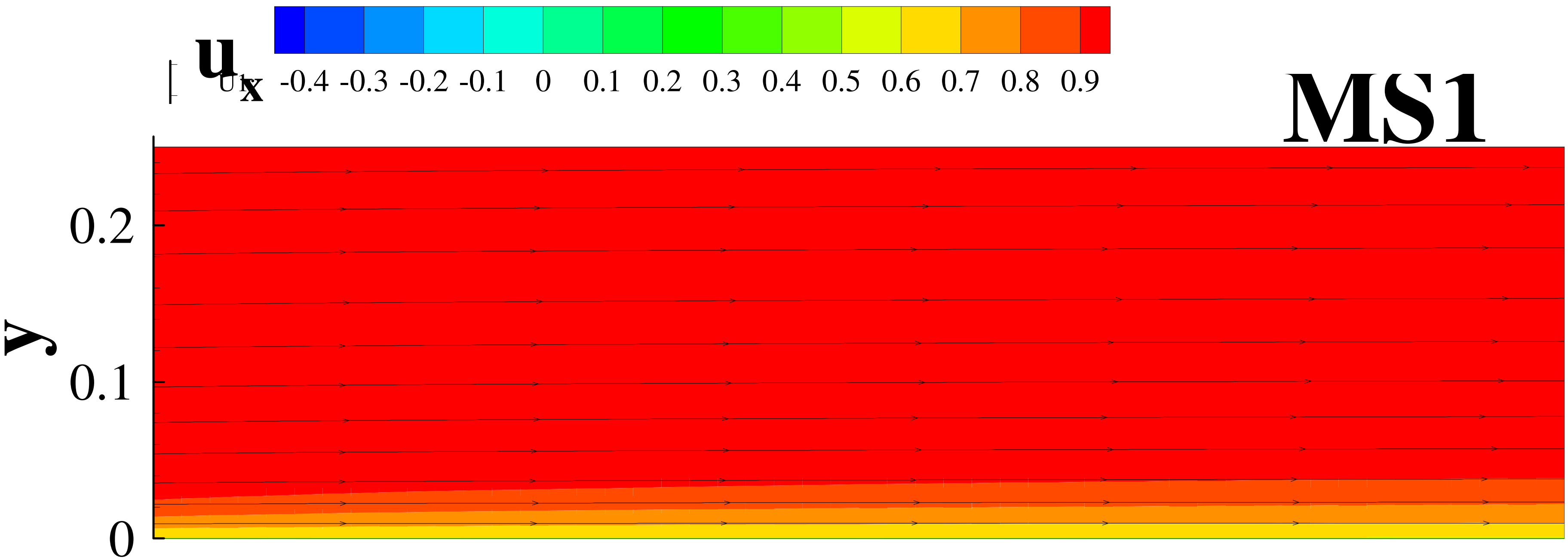
2. Manufactured Solutions

- Turbulence quantities are defined from available expressions for "automatic wall functions" combined with an exponential decay in the outer region
- Free-stream values are adjustable
- Supported turbulence quantities:
  $\tilde{\nu}$, k, ω and Φ
- Alternative functions defined for k, ω

2. Manufactured Solutions

- Mimic of a flat plate boundary-layer

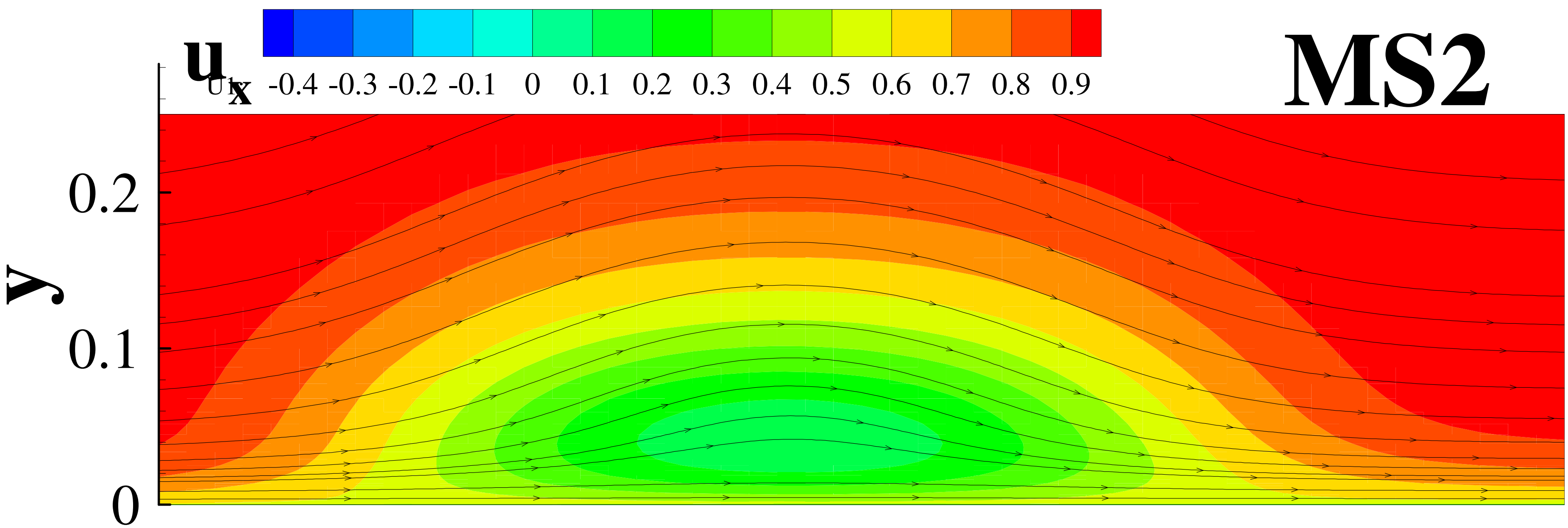
Mean flow field $Re = 10^7$



2. Manufactured Solutions

- Flow with a “weak perturbation”

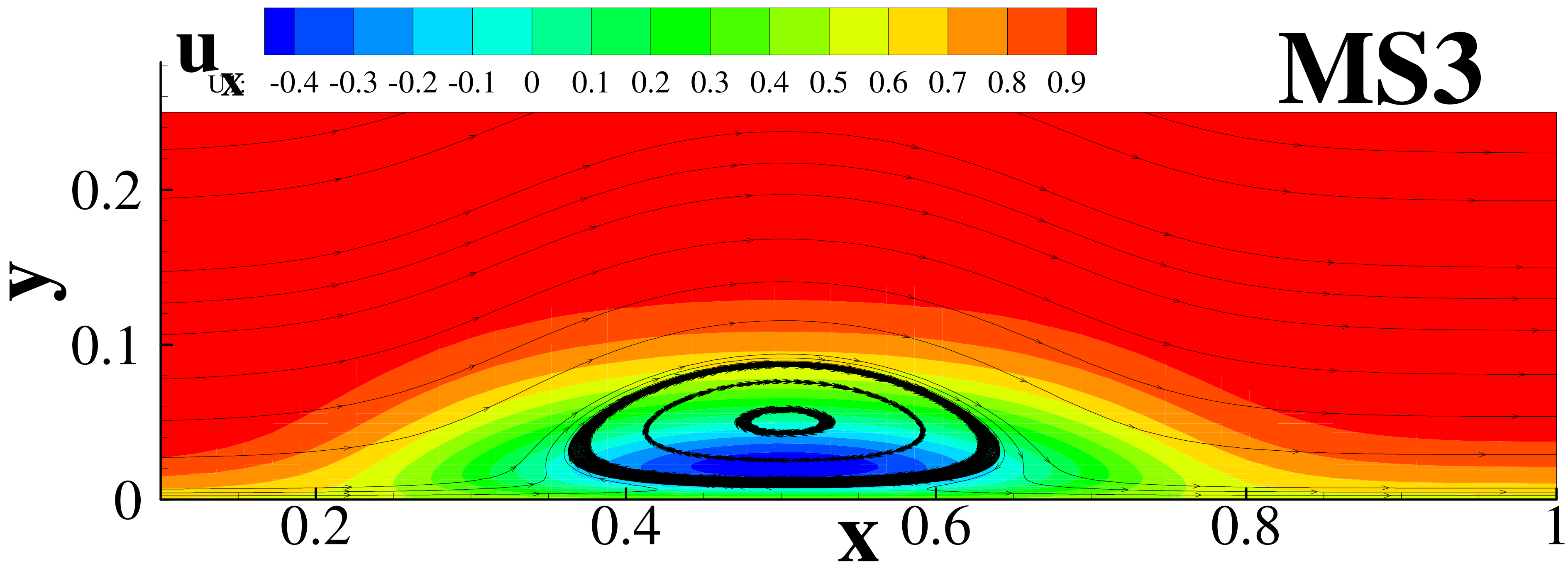
Mean flow field $Re = 10^7$



2. Manufactured Solutions

- Flow with a “strong perturbation”

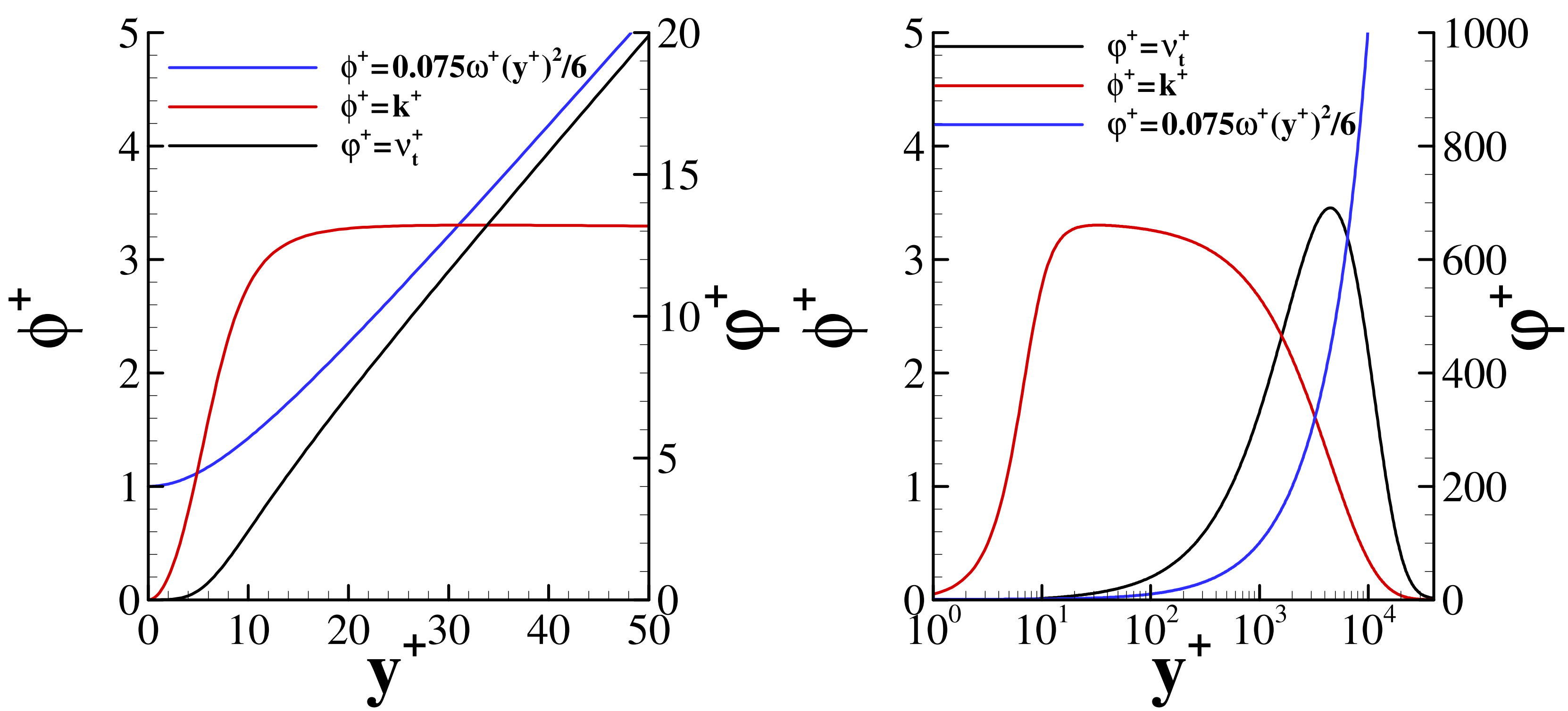
Mean flow field $Re = 10^7$



2. Manufactured Solutions

- Mimic of a flat plate boundary-layer

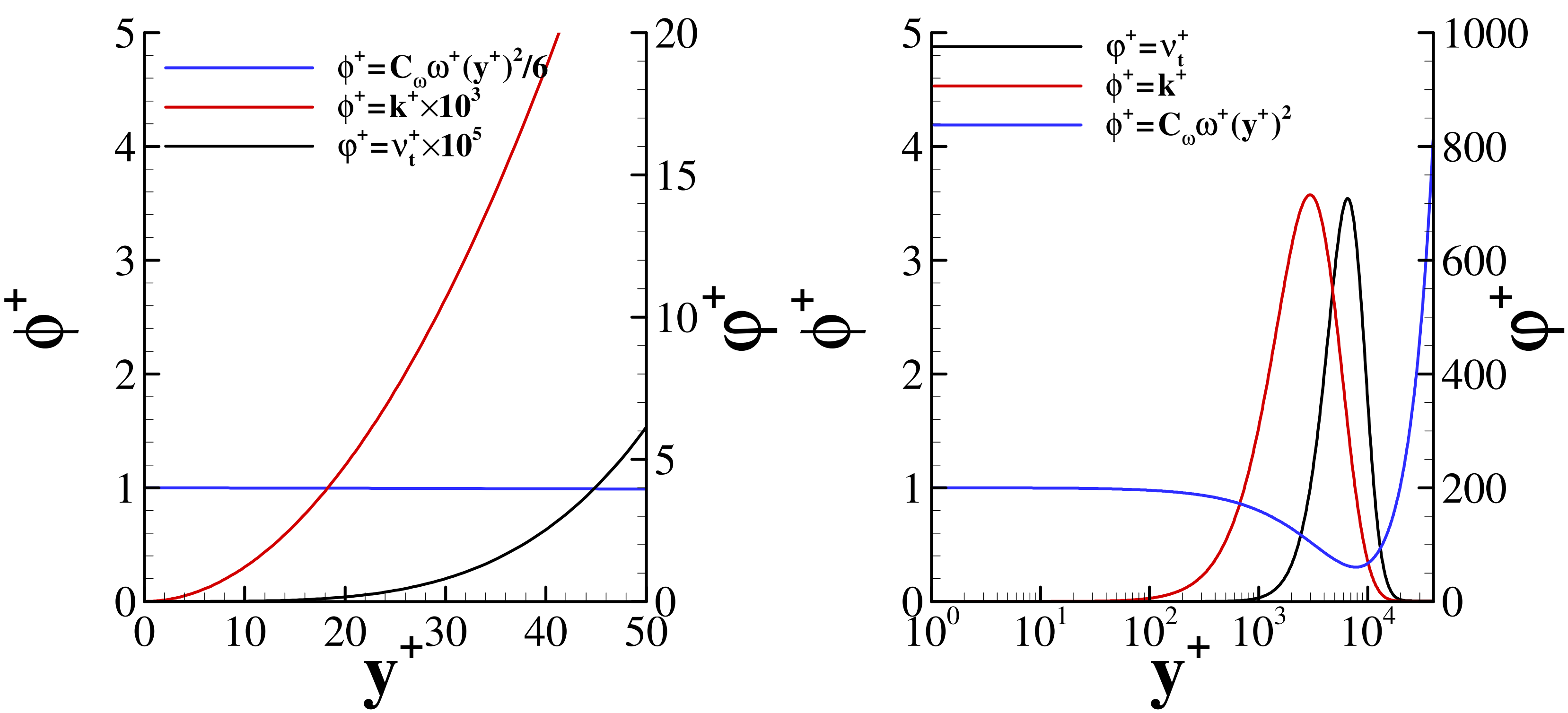
Turbulence quantities $Re = 10^7$



2. Manufactured Solutions

- Alternative definition of k, ω MSa

Turbulence quantities $Re = 10^7$



3. Flow Solvers

- PARNASSOS
  - Finite differences
  - Non-orthogonal, curvilinear, structured grids
  - Coupled solution (momentum and continuity)
  - Continuity solved in its original form $\vec{\nabla} \cdot \vec{V} = 0$

- ReFRESCO
  - Finite volumes
  - Face-based volumes of arbitrary shape
  - Segregated solution
  - SIMPLE-like solution of continuity equation

4. Grid Sets

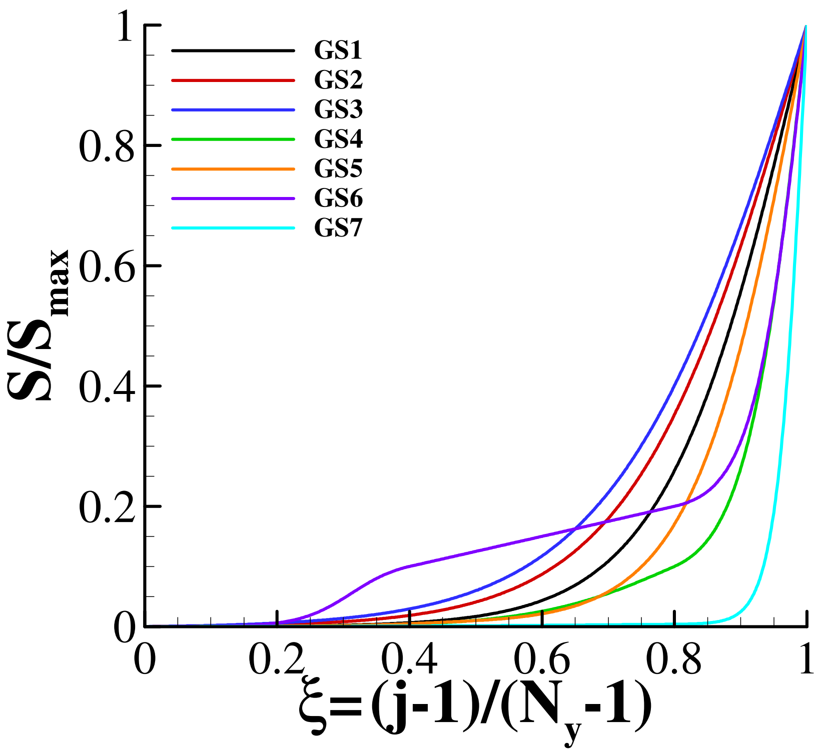
- Sets of 21 geometrically similar stretched Cartesian grids, (51×51 to 801×801)
- Different stretching functions tested with similar near-wall spacing (y_2^+) or same stretching function with different near-wall grid line spacings
- L_∞, L_2 and L_1 norms of the errors of u_x, u_y, C_p, $\nu_t, \tilde{\nu}, k, \omega y^2$

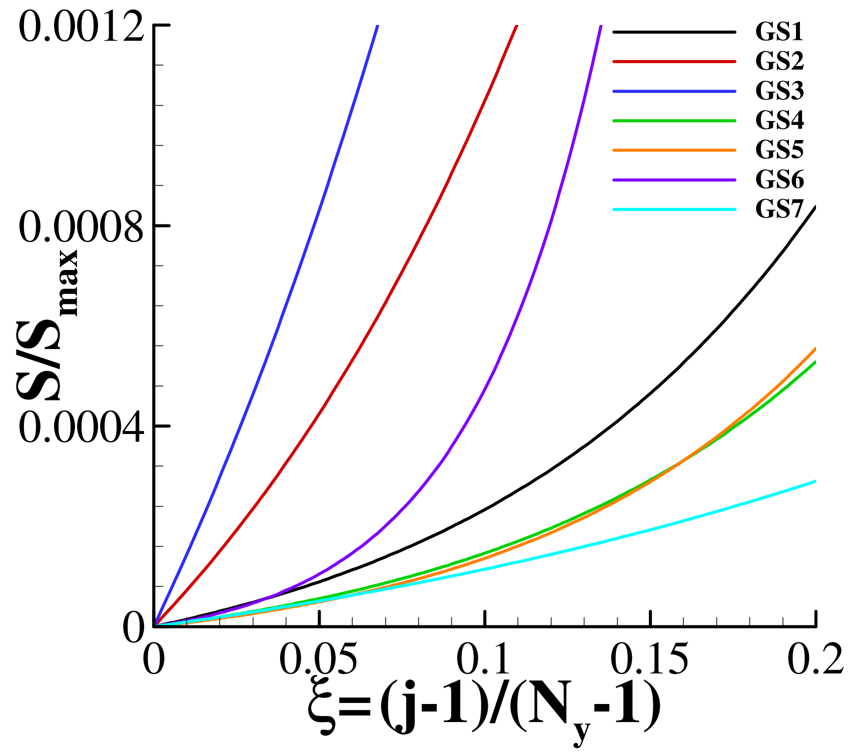
$$e(\phi) = \phi_i - \phi_{exact} \cong e_o + \alpha h_i^p$$

- $e_o(\phi)$, p and α obtained in the least squares sense from the data of the six finest grids

4. Grid Sets

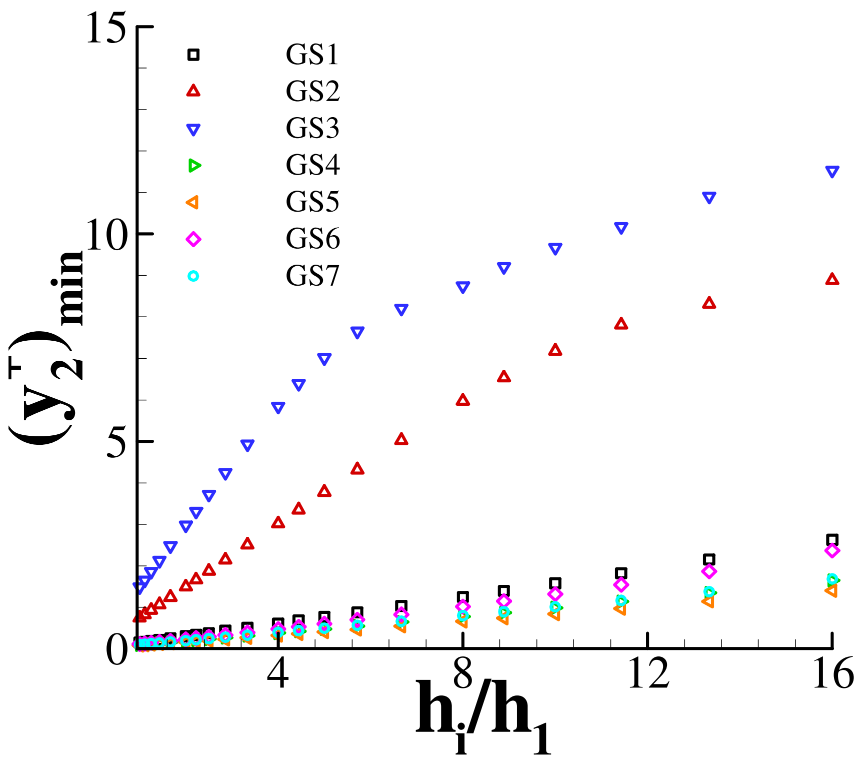
- GS1, GS2 and GS3 – Different y_2^+
- GS1, GS4, GS5, GS6 and GS7 – Different stretching function

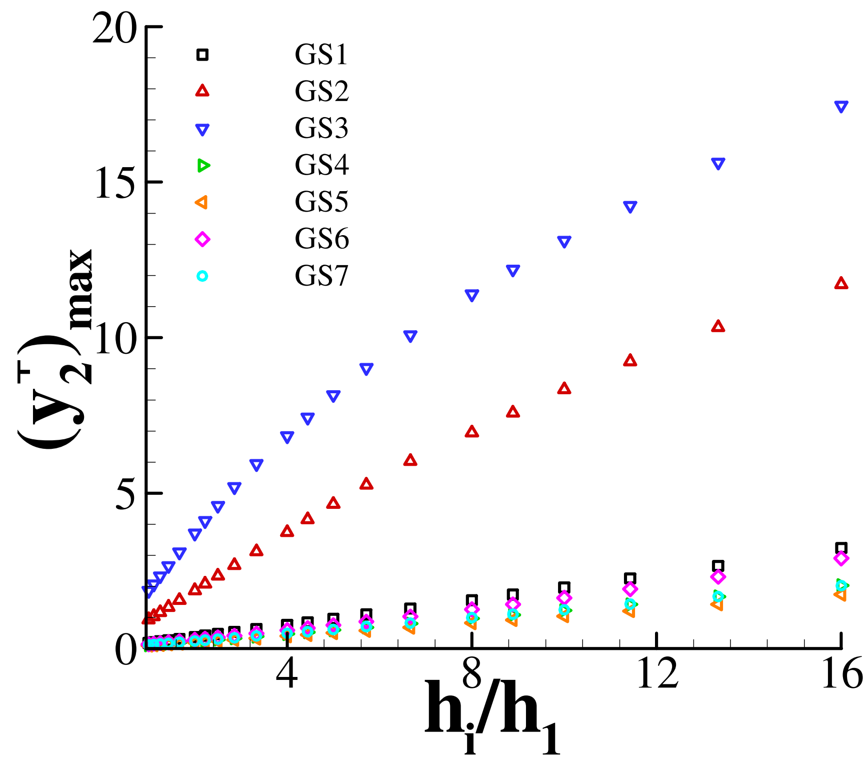




4. Grid Sets

- GS1, GS2 and GS3 – Different y_2^+
- GS1, GS4, GS5, GS6 and GS7 – Different stretching function



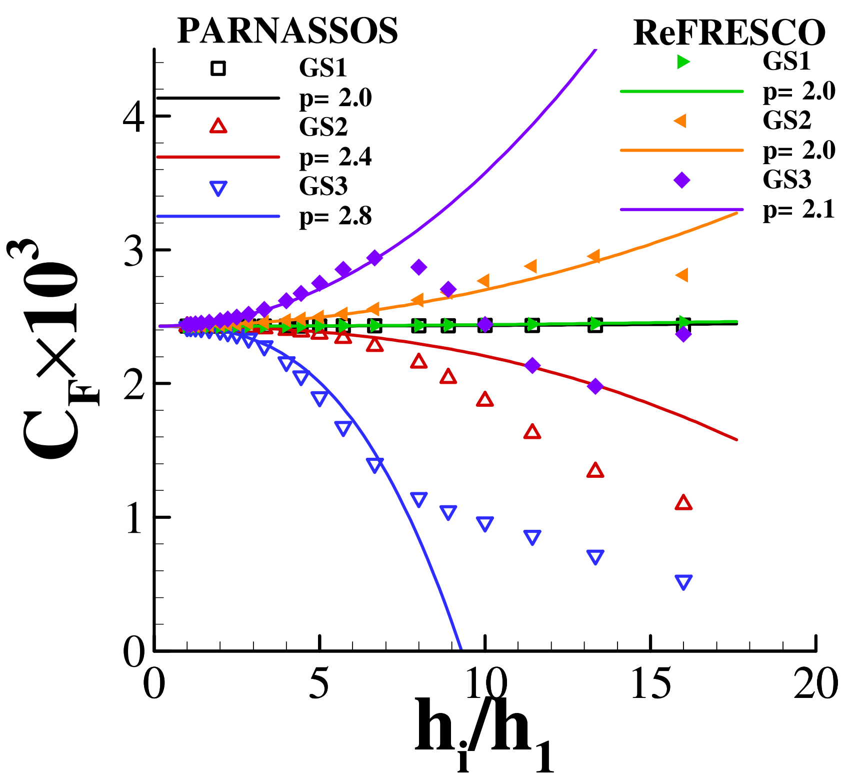


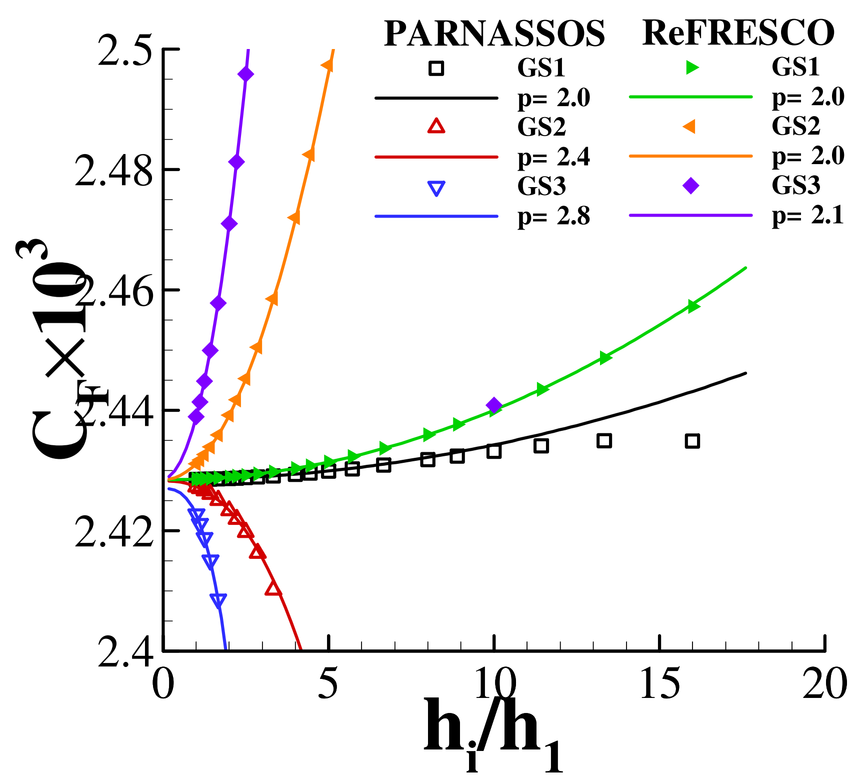
5. Results

- Calculations with manufactured eddy-viscosity field MSt (PARNASSOS, ReFRESCO)
- Calculations with manufactured mean velocity field MSm (PARNASSOS)
- Calculations of all equations with two turbulence models: Spalart & Allmars (SPAL), TNT k-ω (PARNASSOS)
- Friction resistance coefficient, C_F, error norms of $u_x, \nu_t, k, \omega y^2$

5. Results

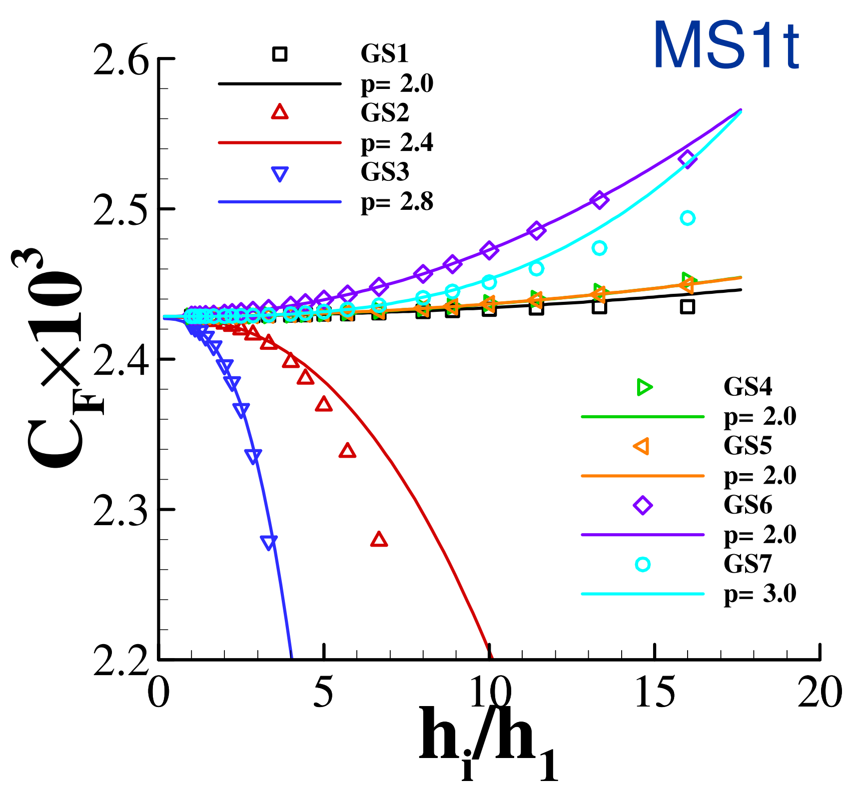
- Friction resistance coefficient, C_F
  Sets with different y_2^+, MS1t

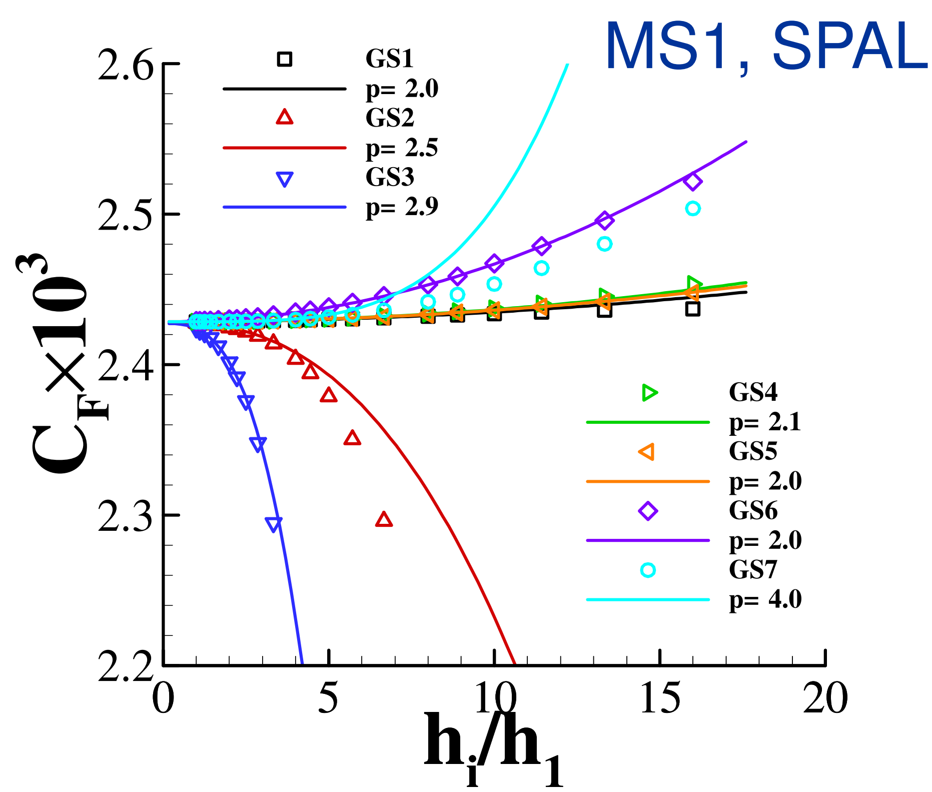




5. Results

- Friction resistance coefficient, C_F
  All sets, PARNASSOS

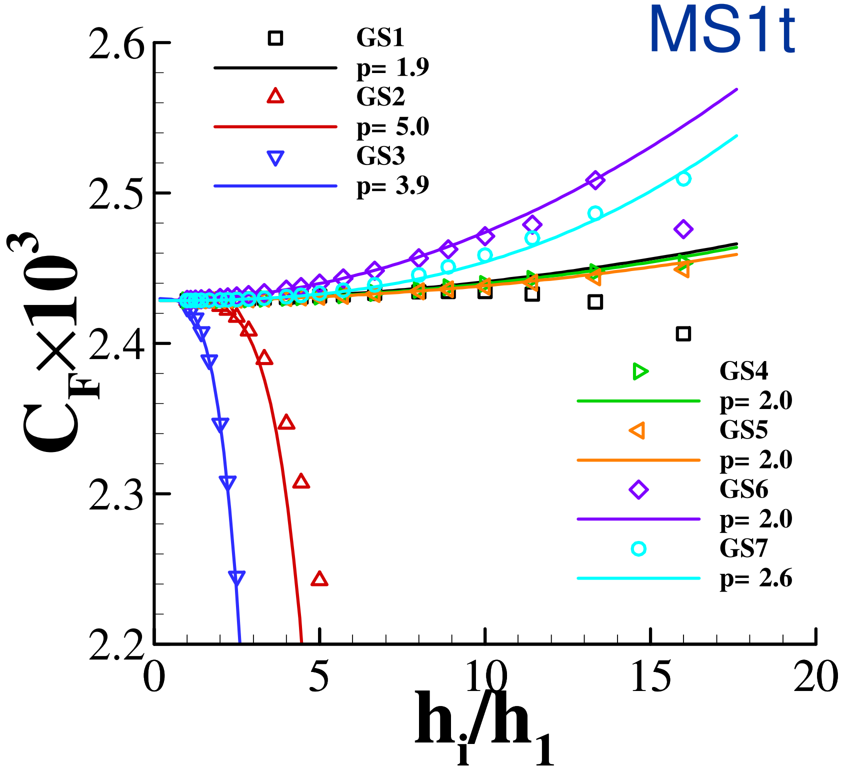




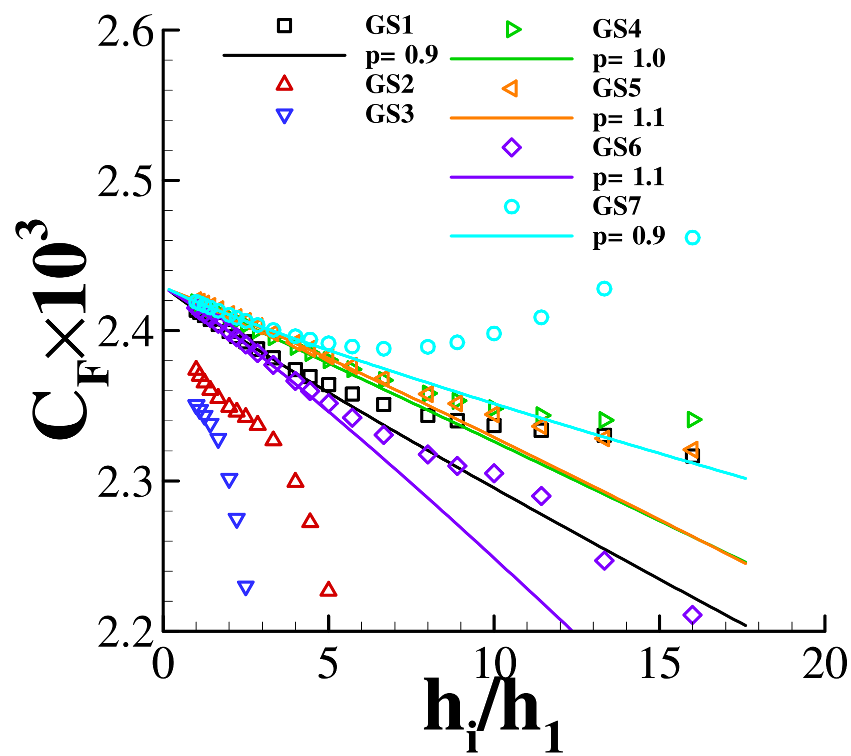
5. Results

- Friction resistance coefficient, C_F
  All sets, PARNASSOS

MS1t



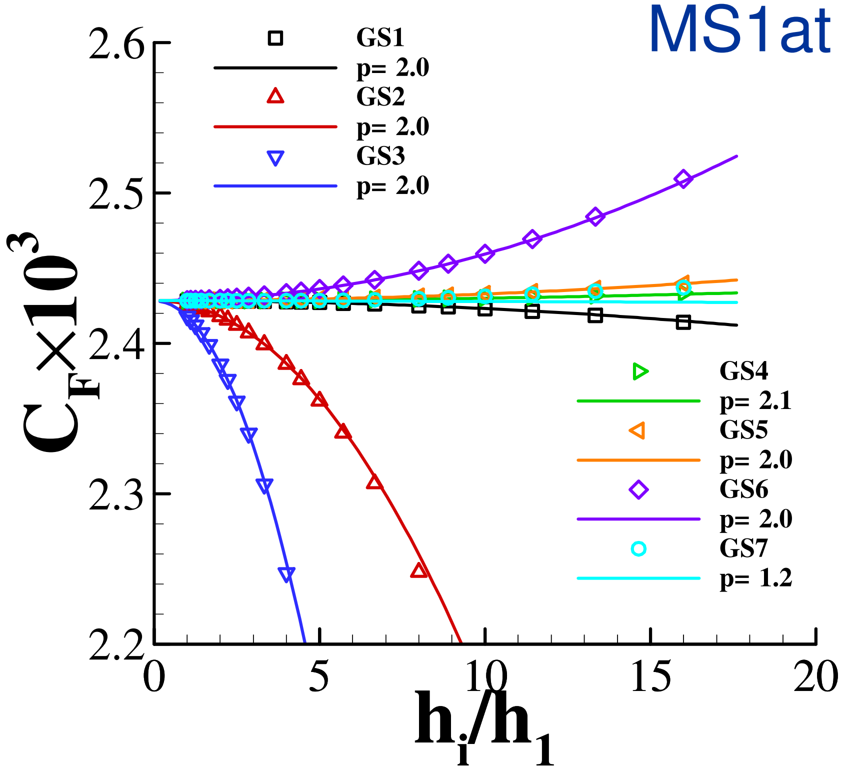
MS1, TNT



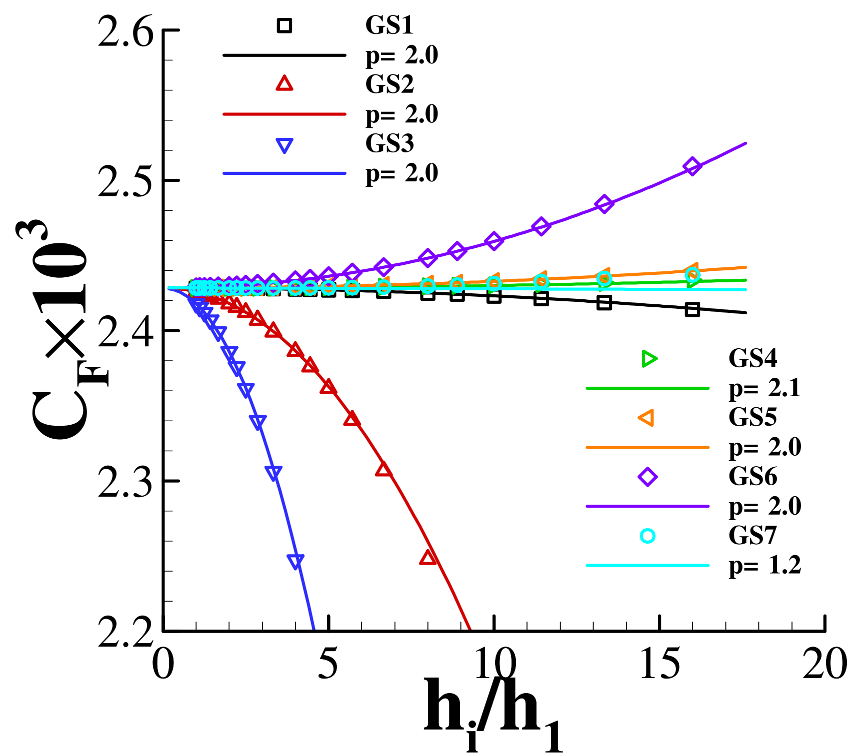
5. Results

- Friction resistance coefficient, C_F
  All sets, PARNASSOS

MS1at

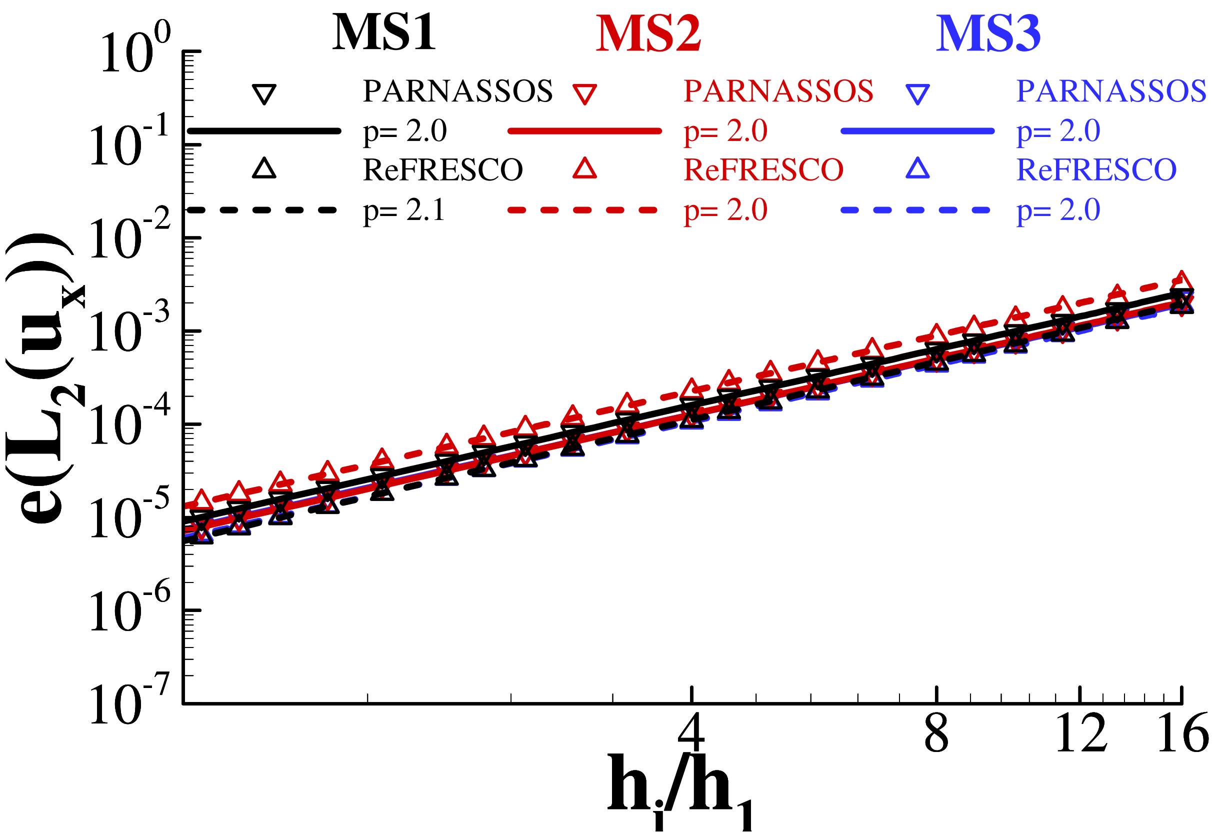


MS1a, TNT



5. Results

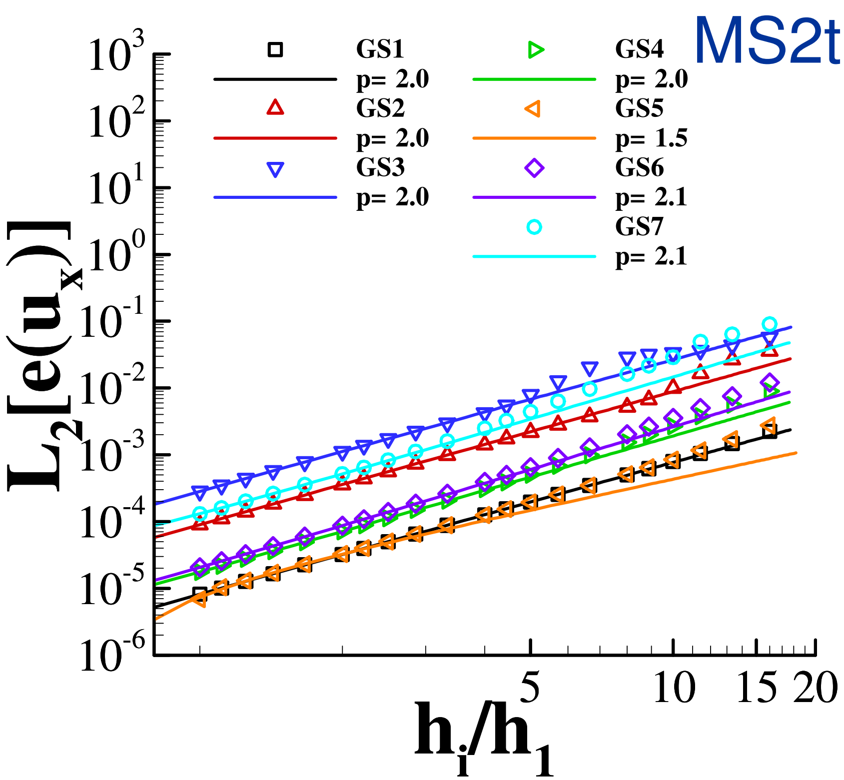
- Horizontal velocity component, u_x
  GS1 set, different MSt

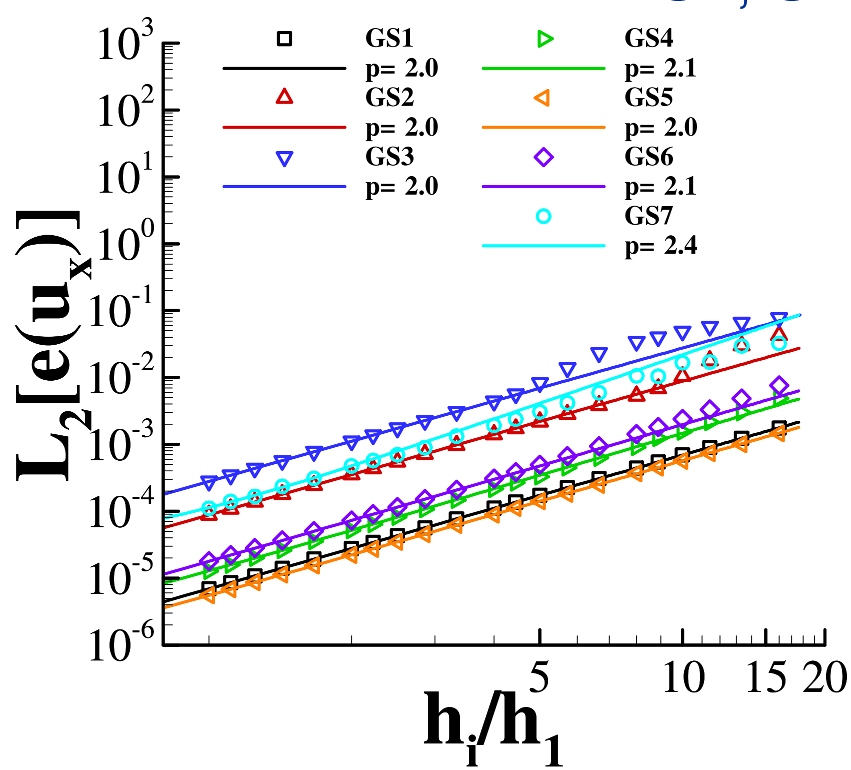


5. Results

- Horizontal velocity component, u_x
  All sets, PARNASSOS

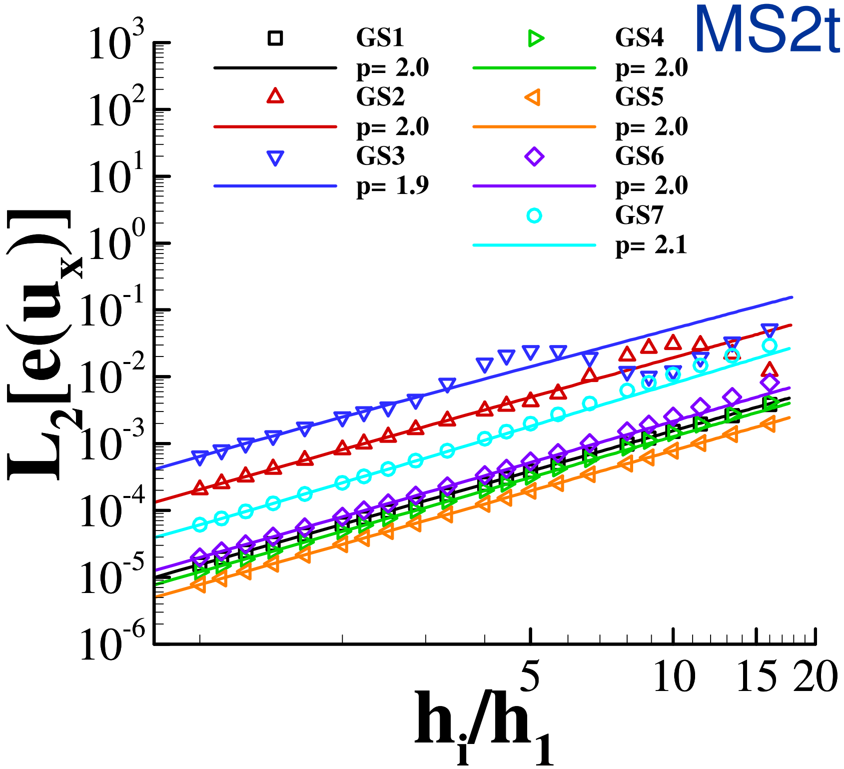
MS2, SPAL

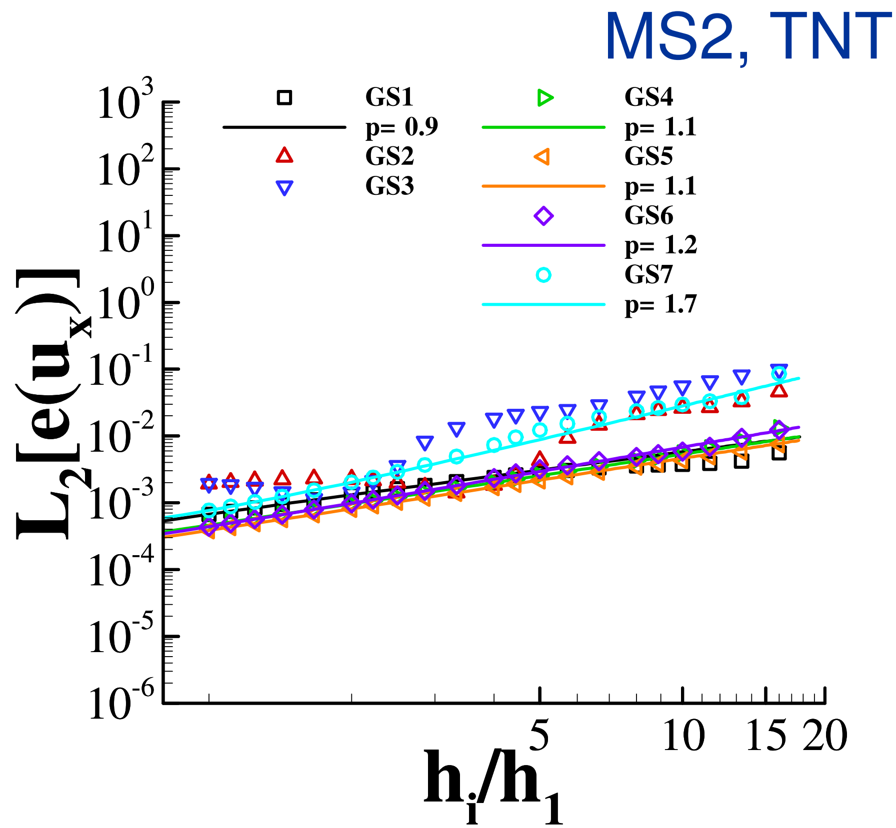




5. Results

- Horizontal velocity component, u_x
  All sets, PARNASSOS

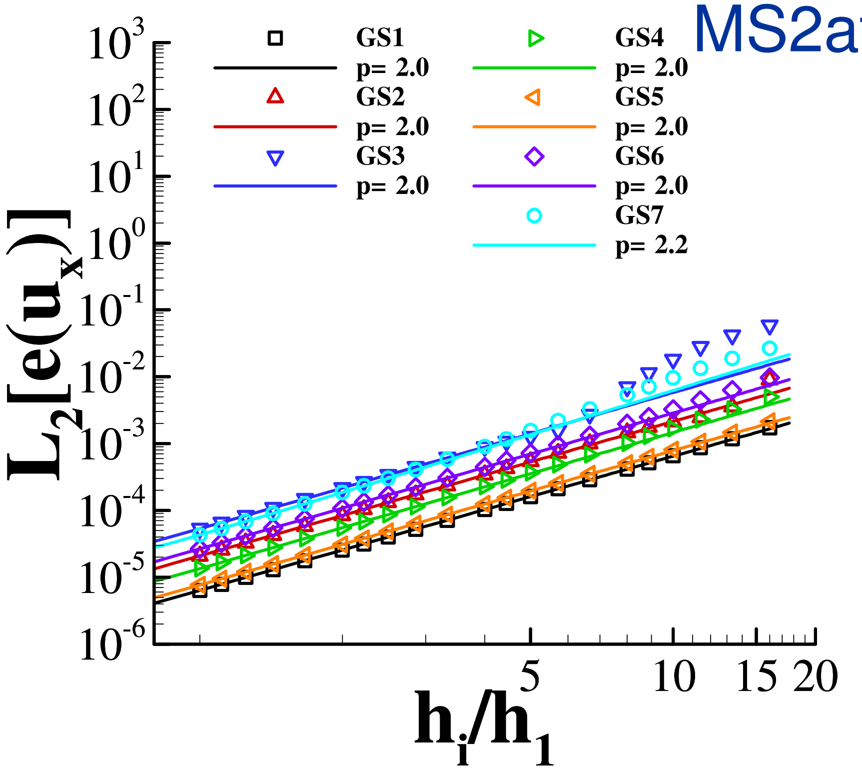




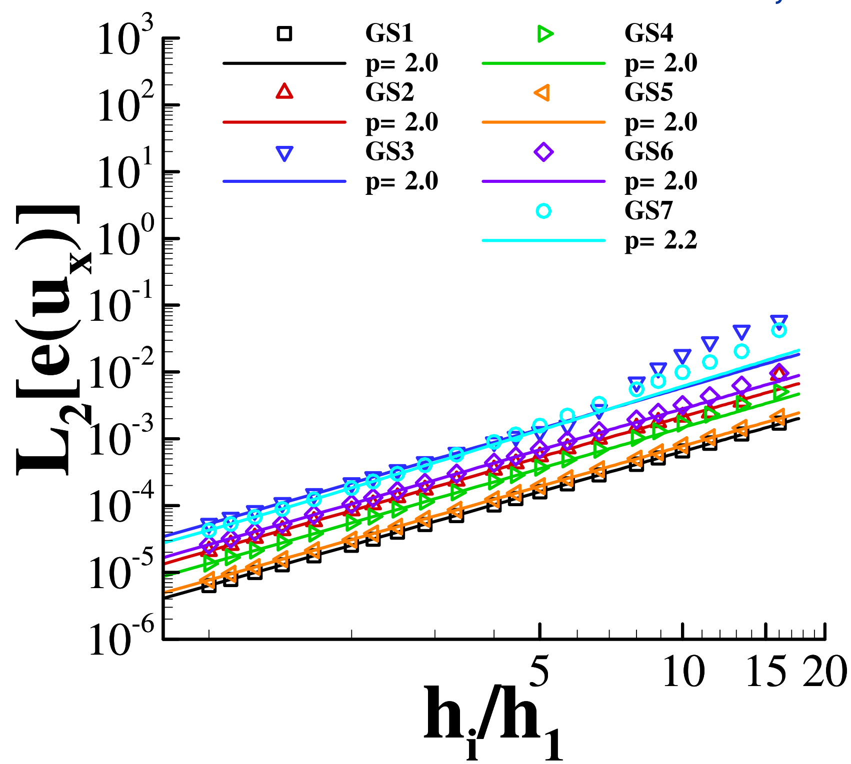
5. Results

- Horizontal velocity component, u_x
  All sets, PARNASSOS

MS2at



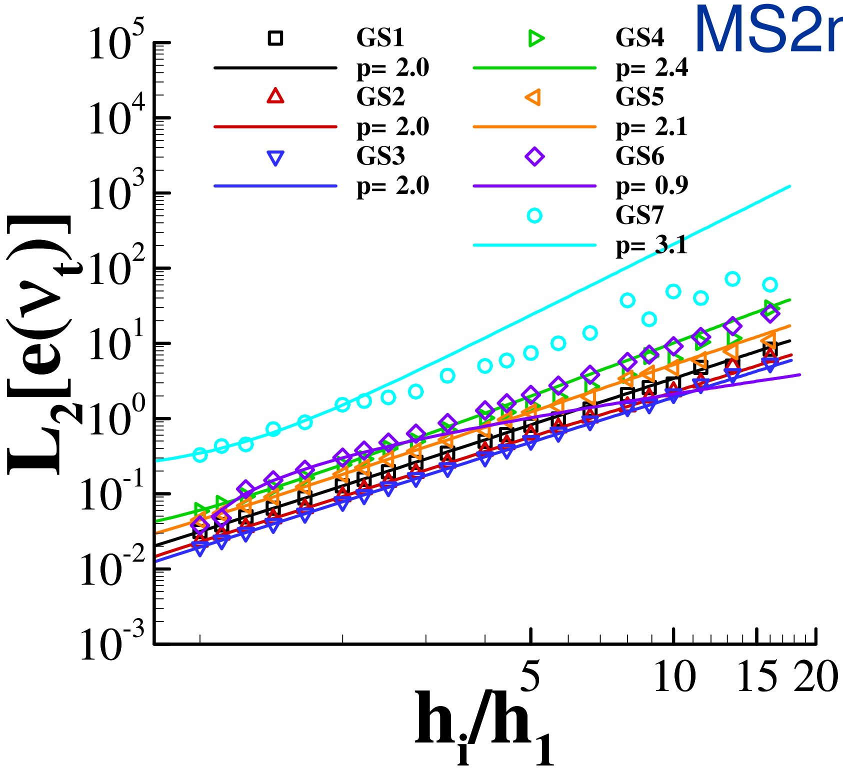
MS2a, TNT



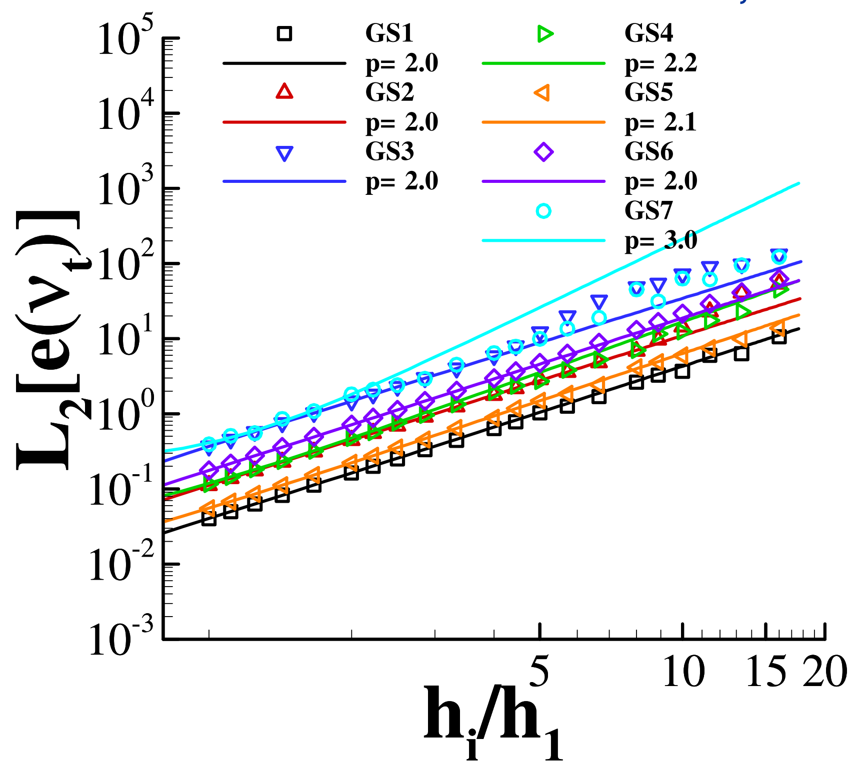
5. Results

- Eddy-viscosity, ν_t
  All sets, PARNASSOS

MS2m

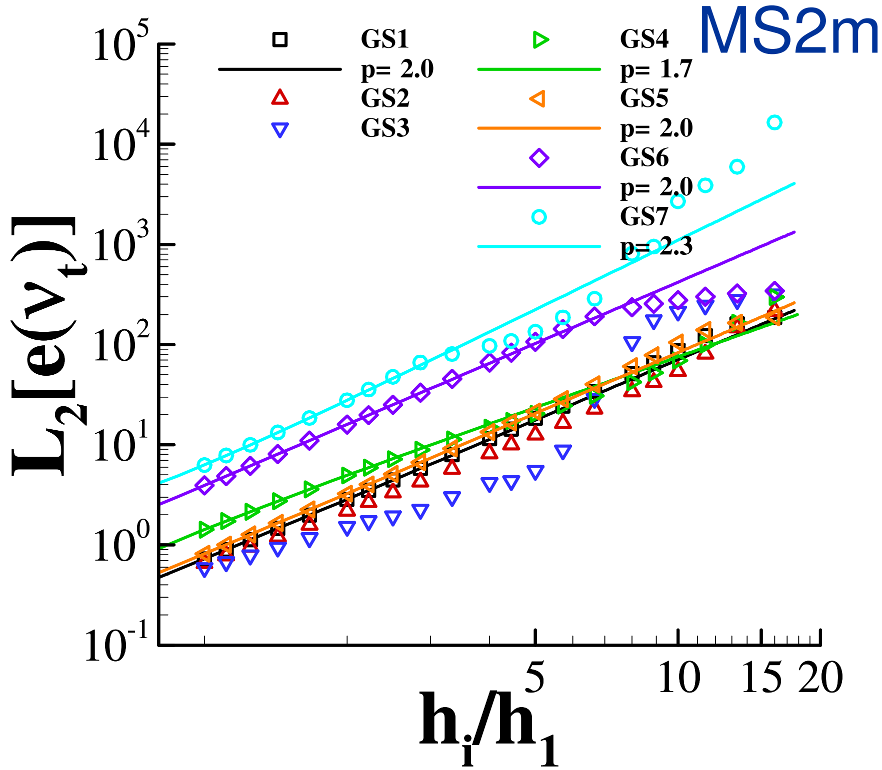


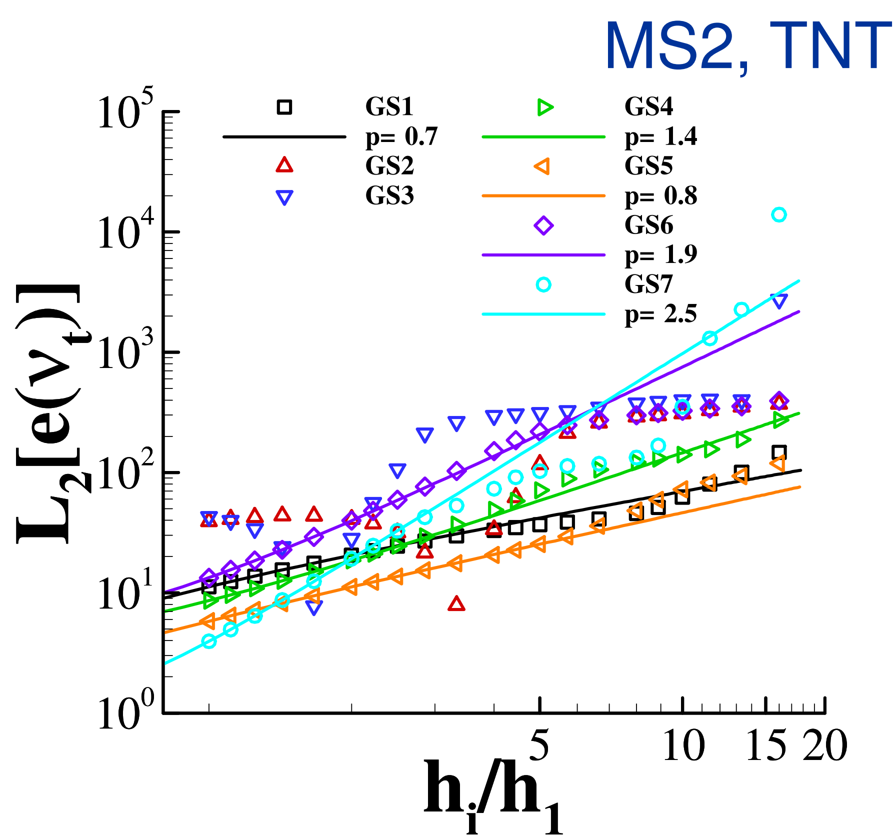
MS2, SPAL



5. Results

- Eddy-viscosity, ν_t
  All sets, PARNASSOS

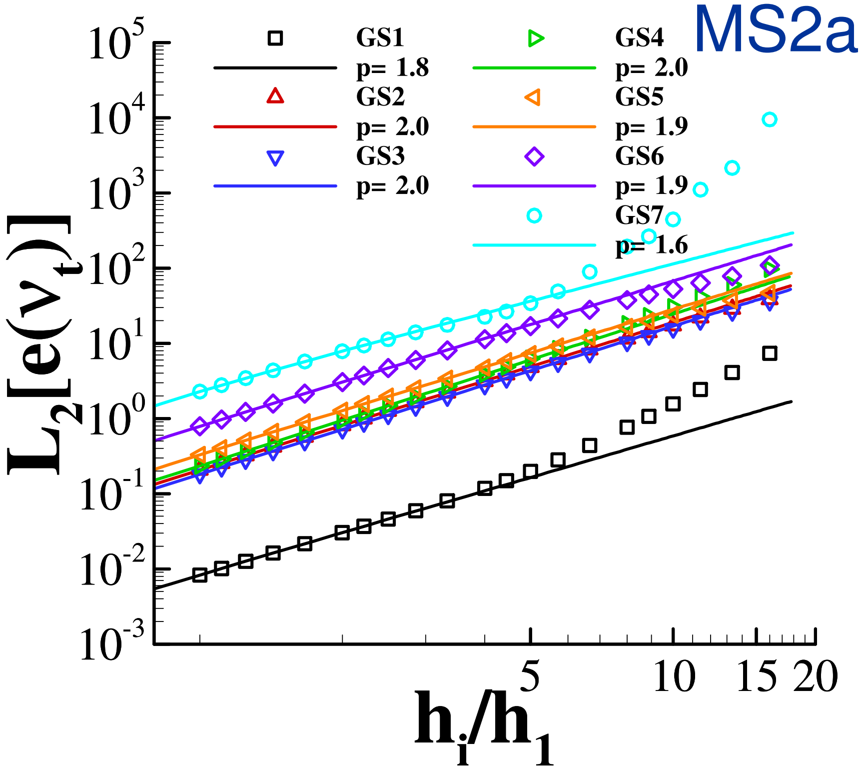




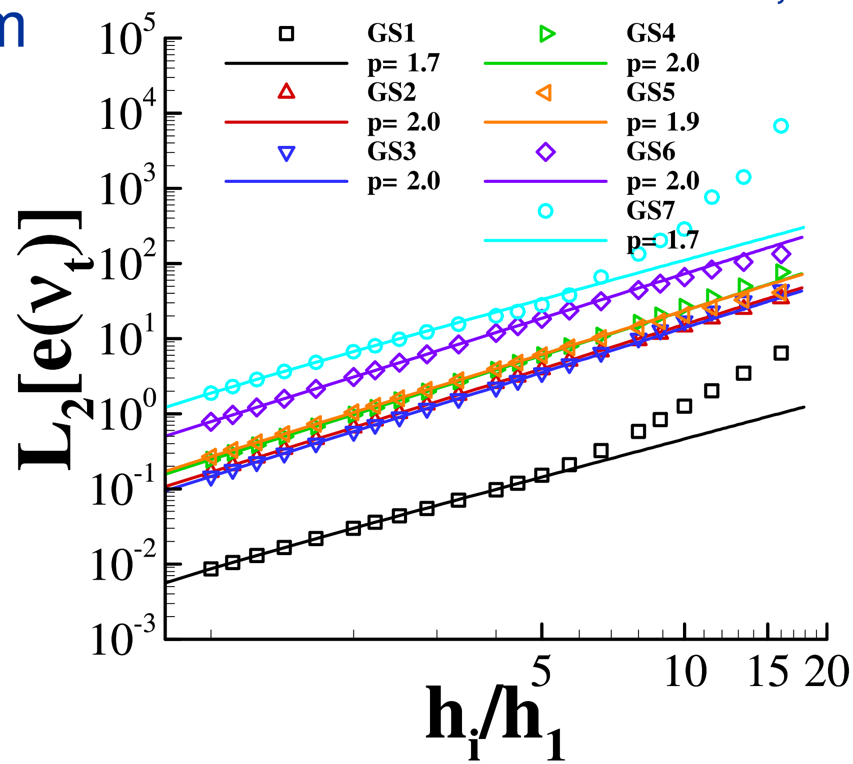
5. Results

- Eddy-viscosity, ν_t
  All sets, PARNASSOS

MS2am



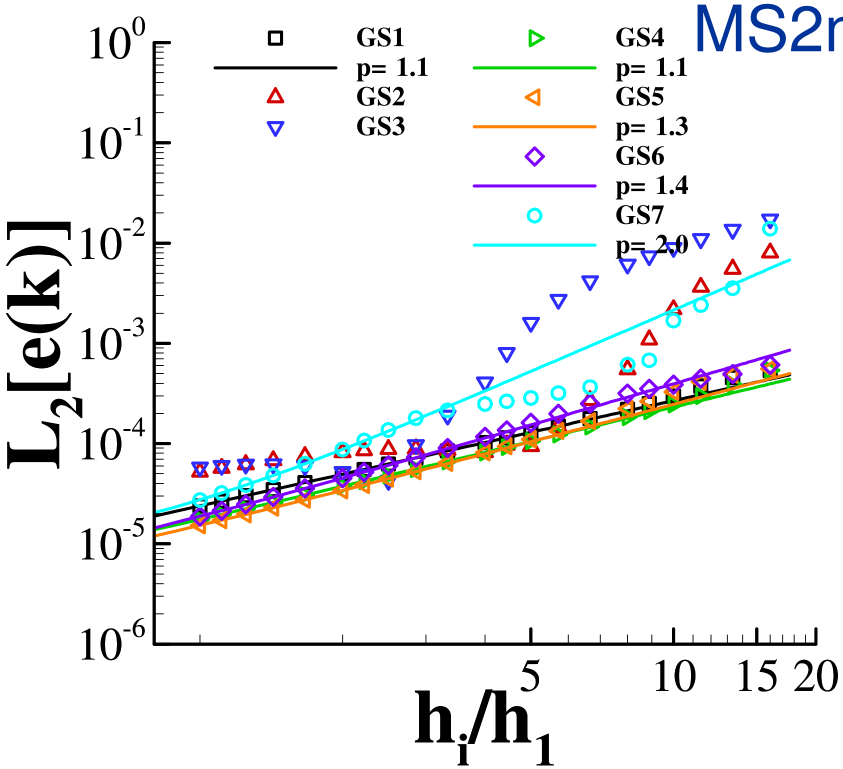
MS2a, TNT



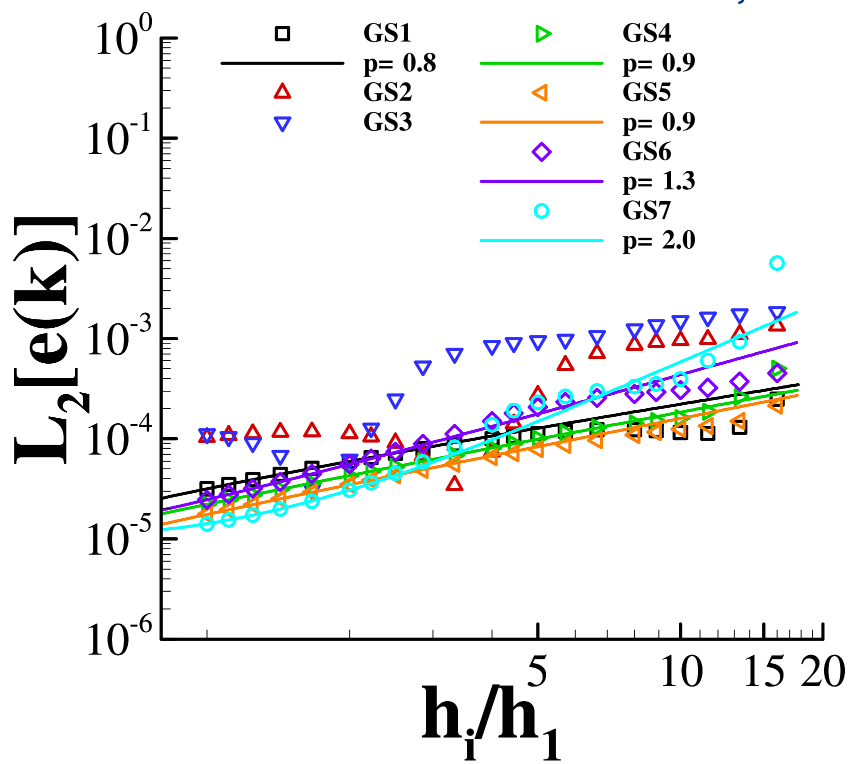
5. Results

- Turbulence kinetic energy, *k*
  All sets, PARNASSOS

MS2m



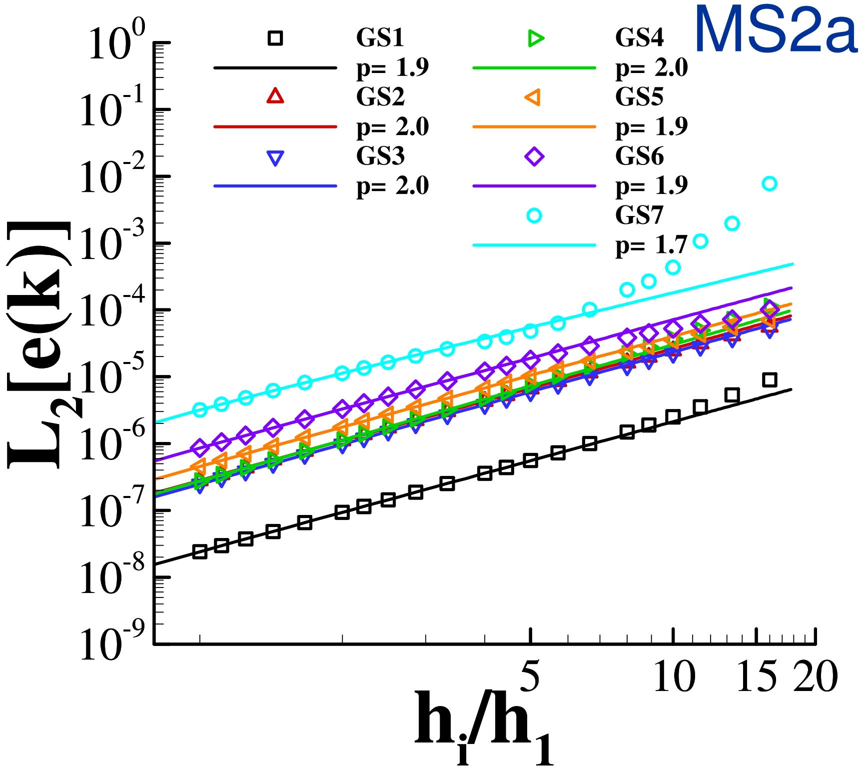
MS2, TNT



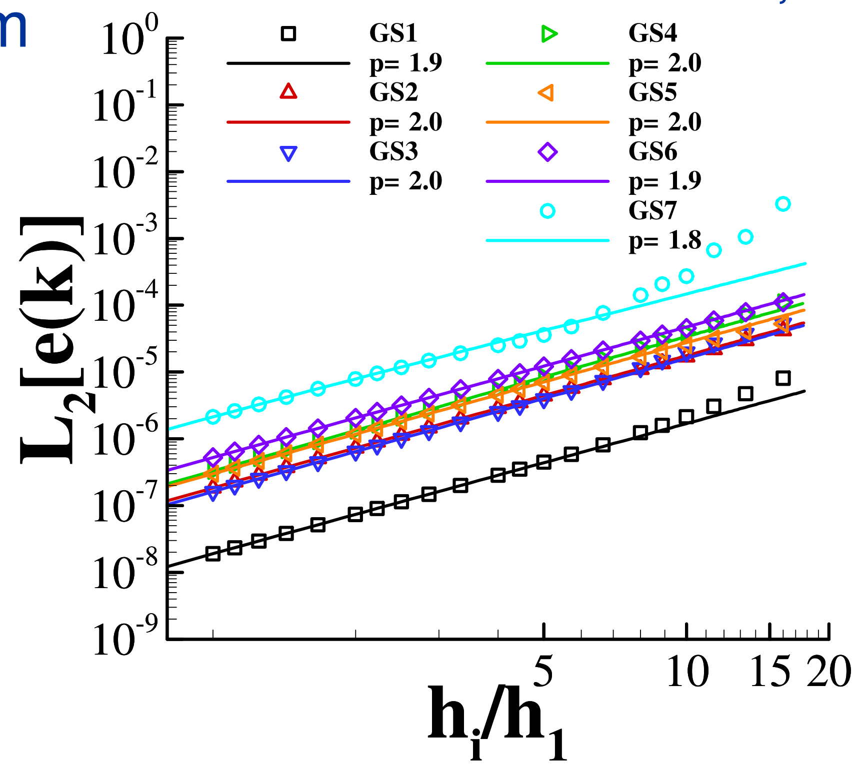
5. Results

- Turbulence kinetic energy, *k*
  All sets, PARNASSOS

MS2am



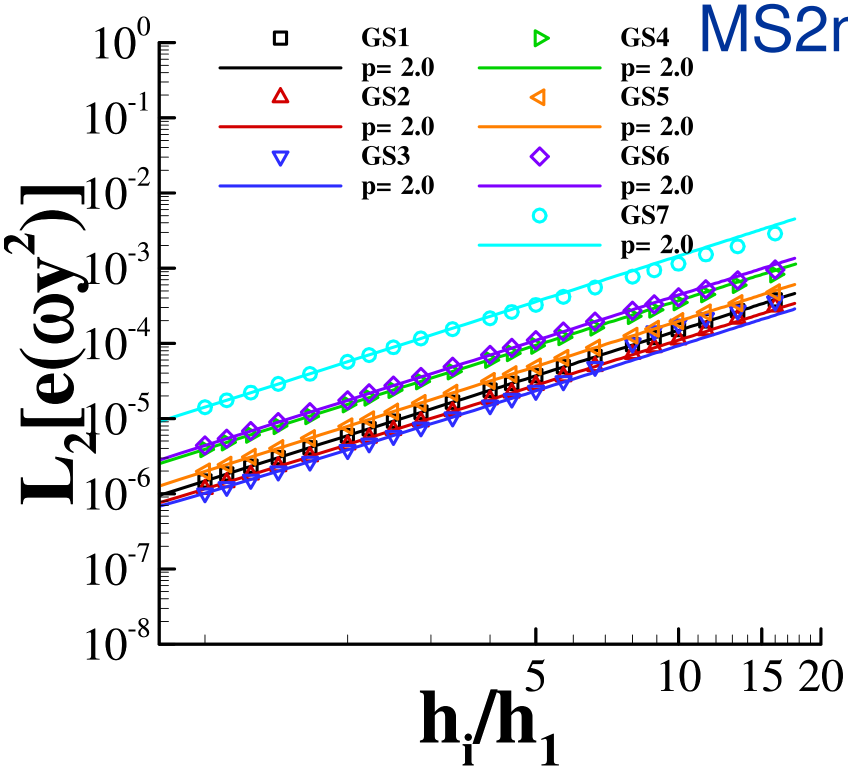
MS2a, TNT



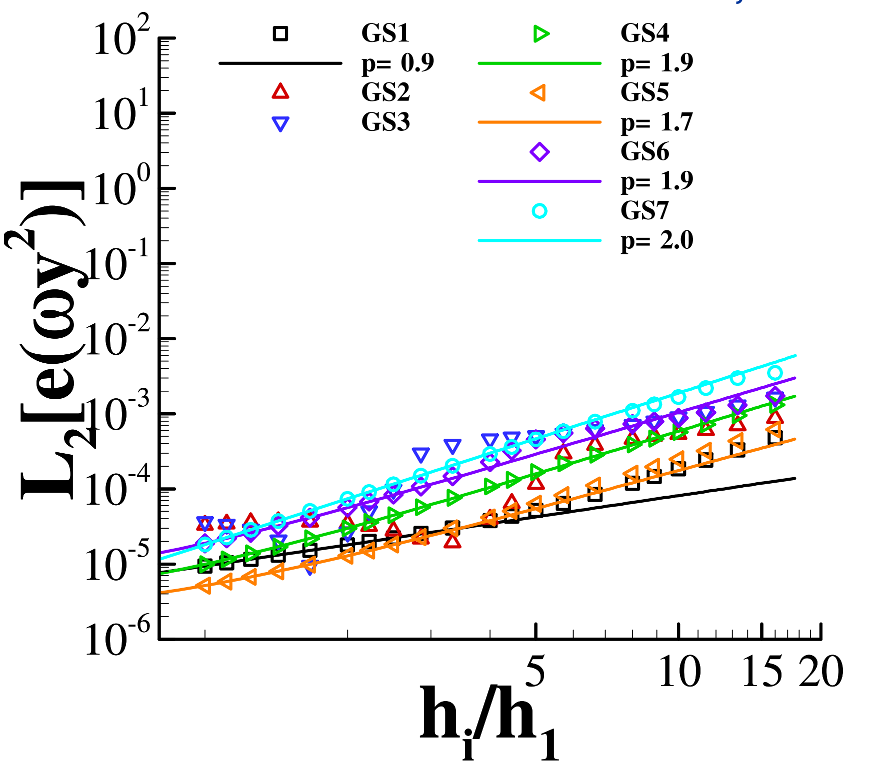
5. Results

- Turbulence frequency, ωy^2
  All sets, PARNASSOS

MS2m



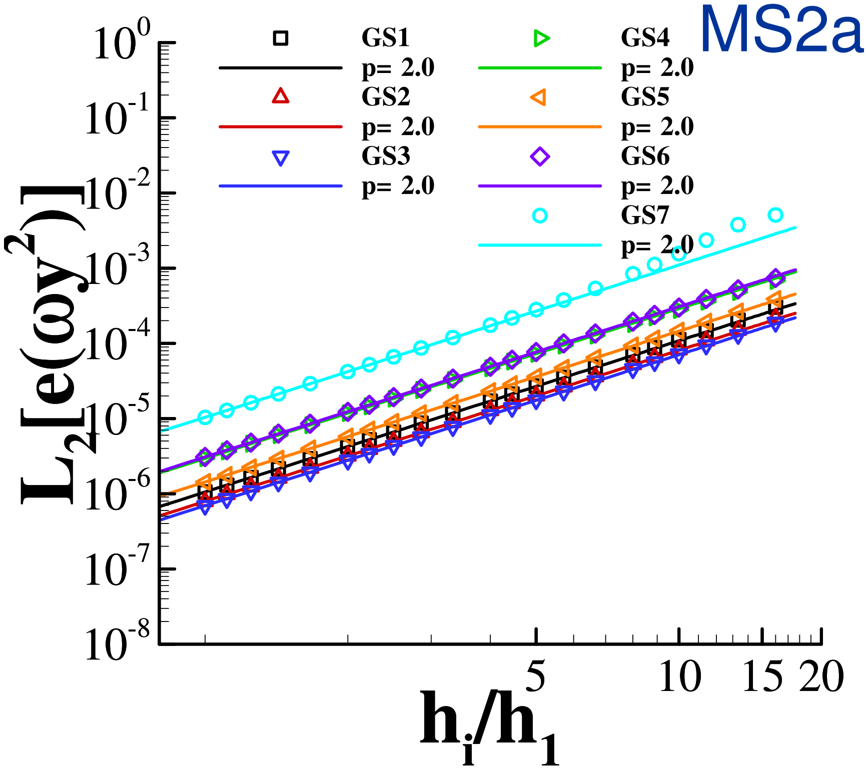
MS2, TNT



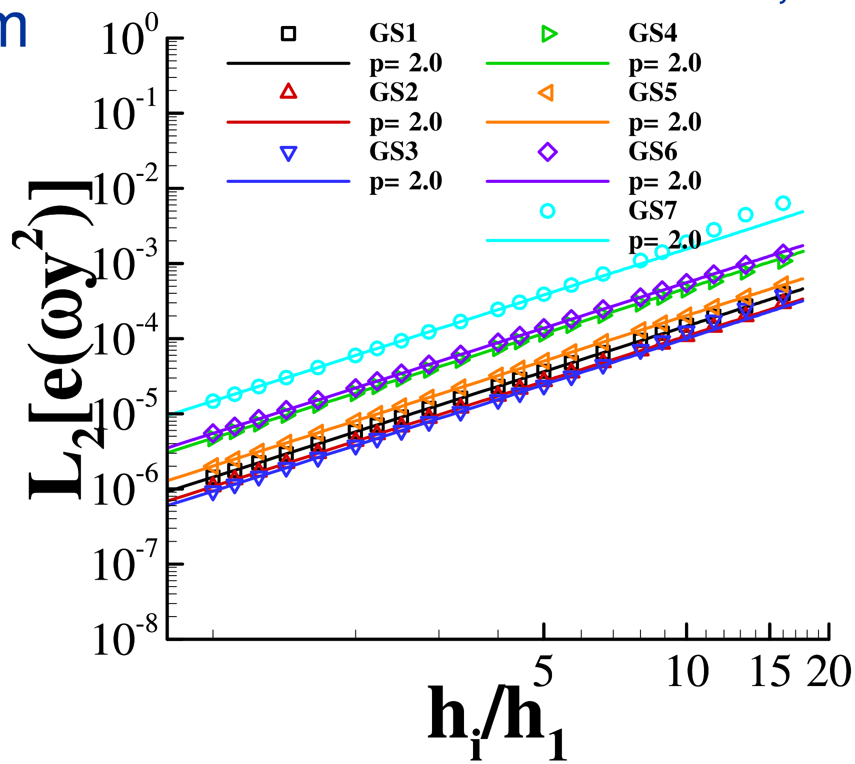
5. Results

- Turbulence frequency, ωy^2
  All sets, PARNASSOS

MS2am



MS2a, TNT



6. Final Remarks

- Convergence properties with manufactured eddy-viscosity follow the expected behaviour
- Turbulence quantities transport equations do not exhibit the expected properties for all the MS tested
- For the most complicated mean velocity field (MS3) it was not possible to obtain any solution of the turbulence quantities transport equations
- Solutions obtained with the TNT k-ω model are extremely sensitive to the near-wall grid line distribution

6. Final Remarks

- All these manufactured solutions are available to the CFD community!